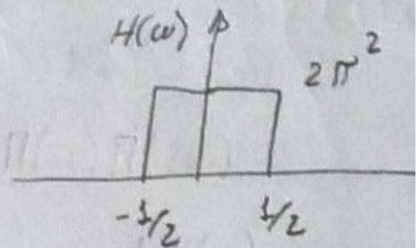
① Desde que $\Pi(t/T) \xleftrightarrow{\mathcal{F}} 2T\,\mathrm{sinc}(t.T)$ e $\mathrm{sinc}(t.T) \xleftrightarrow{\mathcal{F}} \frac{\pi}{T}\Pi$

a)

$$H_1(\omega) = \pi\,\Pi(\omega) \quad e \quad H_2(\omega) = 2\pi\,\Pi(2\omega)$$



$$H(\omega) = \pi\,\Pi(\omega) \times 2\pi\,\Pi(2\omega) = 2\pi^2\,\Pi(2\omega)$$

$$H(\omega) = \begin{cases} 2\pi^2, & |\omega| \leq 1/2 \\ 0, & cc \end{cases}$$

b) A saída é $y(t) = x(t) * h(t)$ e $Y(\omega) = X(\omega) \cdot H(\omega)$

como $X(\omega) = \frac{d}{d\omega}(\cdot)$ $\therefore$ $Y(\omega) = \frac{d}{d\omega} H(\omega)$

$$Y(\omega) = \begin{cases} 2\pi^2, & \omega = -1/2 \\ -2\pi^2, & \omega = 1/2 \\ 0, & cc. \end{cases}$$

② a) $H(\omega) = \dfrac{j\omega + 2}{(j\omega)^2 + 4j\omega + 3} = \dfrac{j\omega + 2}{(j\omega + 3)(j\omega + 1)} = \dfrac{A_1}{j\omega + 3} + \dfrac{A_2}{j\omega + 1} = \dfrac{1/2}{j\omega + 3} + \dfrac{1/2}{j\omega \ldots}$

$$A_1 = \frac{-3+2}{-3+1} = \frac{1}{2}, \quad A_2 = \frac{-1+2}{-1+3} = \frac{1}{2}$$

$$h(t) = \left(\frac{1}{2}e^{-3t} + \frac{1}{2}e^{-t}\right)u(t)$$

b) $H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = \dfrac{j\omega + 2}{(j\omega)^2 + 4j\omega + 3} \Rightarrow (j\omega)^2 Y(\omega) + 4j\omega Y(\omega) + 3Y(\omega) = j\omega X(\omega) + 2 \ldots$

$$\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 3y(t) = \frac{d}{dt}x(t) + 2x(t)$$

③ • Se $x(t)$ é real e par, então os polos são simétricos e conjugados

• $X(s) = \dfrac{K}{(s \pm a)(s \pm a^*)}$

• Se $a = 2e^{-j\pi/4}$ $\Rightarrow X(s) = \dfrac{K}{(s+2e^{j\pi/4})(s+2e^{-j\pi/4})(s-2e^{j\pi/4})(s-2e^{-j\pi/4})}$

• Se $\int x(t)\,dt = \frac{1}{4} \Rightarrow X(0) = \int_{-\infty}^{\infty} x(t)e^{0.t}\,dt = \frac{1}{4}$

$$X(s) = \frac{K}{(s^2+2\sqrt{2}s+4)(s^2-2\sqrt{2}s+4)} \quad \therefore X(0) = \frac{1}{4} = \frac{K}{4.4} \Rightarrow K = 4$$

$$\boxed{X(s) = \frac{4}{(s^2+2\sqrt{2}s+4)(s^2-2\sqrt{2}s+4)}, \; -\sqrt{2} < Re\{s\} < 2}$$

④ $H(s) = \dfrac{(s-1)(s-3)}{(s+2)(s+1)^2}$

zeros $s = \{1, 3, \infty\}$
polos $s = \{-1, -2\}$ dois polos em -1

$$H(s) = \frac{A_1}{s+2} + \frac{A_2}{s+1} + \frac{A_3}{(s+1)^2} = \frac{(s+1)^2A_1 + (s+1)(s+2)A_2 + (s+2)A_3}{(s+2)(s+1)^2} = \frac{s^2-4s+3}{(s+2)(s+1)^2}$$

$$H(s) = \frac{15}{s+2} - \frac{14}{s+1} + \frac{8}{(s+1)^2}$$

$$\boxed{h(t) = -15e^{-2t}u(-t) - (14e^{-t} + 8te^{-t})u(t)}$$

$$\begin{cases} ① & A_1 + A_2 = 1 \\ ② & 2A_1 + 3A_2 + A_3 = -4 \\ ③ & A_1 + 2A_2 + 2A_3 = 3 \end{cases}$$

② - ① - ③

$0A_1 + 0A_2 - A_3 = -4 - 1 - 3$

$A_3 = 8$

③ - ① $A_2 + 2A_3 = 2 \Rightarrow A_2 = 2 - 16 = -14 \Rightarrow A_1 = 15$