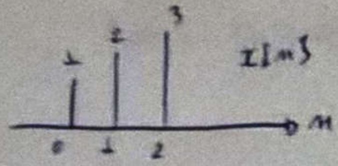
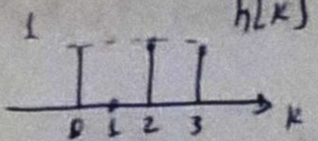
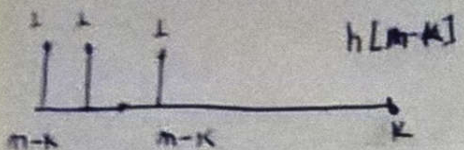
Gabarito Sinais 2ª chamada 2016.2

1)







$$y[n] = x[n] * h[n] = \sum_k x[k] \cdot h[n-k]$$

$y[0] = x[0]h[0] = 1$

$y[1] = x[0]h[1] + x[1]h[0] = 2$

$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0] = 4$

$y[3] = x[0]h[3] + x[1]h[2] + x[2]h[1] = 3$

$y[4] = x[1]h[3] + x[2]h[2] = 5$

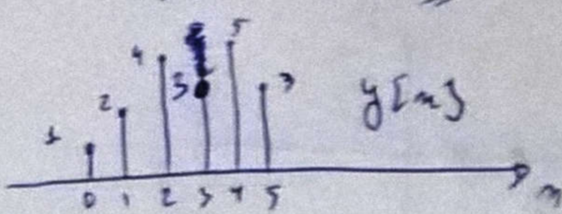
$y[5] = x[2]h[3] = 3$

Início

$n - 0 = 0 \Rightarrow n = 0$

Fim

$n - 3 = 2 \Rightarrow n = 5$



2) a) $X_1(\omega) = \int_{-\infty}^{\infty} e^{-2t} u(t-2) e^{-j\omega t} dt = \int_2^{\infty} e^{-2t} e^{-j\omega t} dt = \frac{-1}{j\omega+2} \cdot e^{-(j\omega+2)t}\Big|_{t=2}^{\infty}$

$X_1(\omega) = \frac{1}{j\omega+2} \cdot e^{-2j\omega-4}$

b) $\mathrm{sinc}(t) \xleftrightarrow{\mathcal{F}} \pi \Pi(\omega) = \begin{cases} 1, & |\omega| \leq 1 \\ 0, & c.c. \end{cases}$

$X_2(\omega) = X_1(\omega)\, \pi \Pi(\omega) = \begin{cases} \pi \cdot X_1(\omega), & |\omega| \leq 1 \\ 0, & c.c. \end{cases} = \begin{cases} \frac{\pi \cdot e^{-2j\omega-4}}{j\omega+2}, & |\omega| \leq 1 \\ 0, & c.c. \end{cases}$

c) $x_2(t)\, e^{j4t} \xleftrightarrow{\mathcal{F}} X_2(\omega-4)$, $\frac{d^3}{dt^3} x(t) \xleftrightarrow{\mathcal{F}} (j\omega)^3 X(\omega)$

$X_3(\omega) = (j\omega)^3 X_2(\omega-4) = -j\pi\omega^3 \begin{cases} X_1(\omega), & |\omega-4| \leq 1 \\ 0, & cc \end{cases}$

$X_3(\omega) = \begin{cases} -j\pi\omega^3 X_1, & 3 \leq \omega \leq 5 \\ 0, & cc \end{cases} = \begin{cases} \frac{-j\pi\omega^3 \cdot e^{-2j\omega-4}}{j\omega+2}, & 3 \leq \omega \leq 5 \\ 0, & cc \end{cases}$

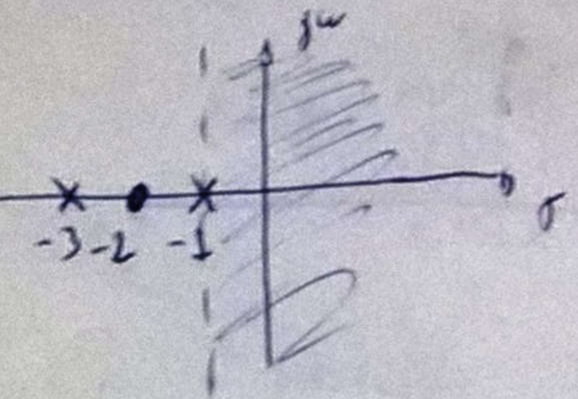
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2 + j\omega}{3 + 4j\omega + (j\omega)^2} \Rightarrow (j\omega)^2 Y(\omega) + 4j\omega Y(\omega) + 3Y(\omega) = j\omega X(\omega) + 2X(\omega)$$

$$\frac{d^2}{dt^2} y(t) + 4\frac{d}{dt} y(t) + 3y(t) = \frac{d}{dt} x(t) + 2x(t)$$

$$\frac{d}{dt} \xleftrightarrow{\mathcal{L}} s$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+2}{s^2 + 4s + 3}$$

Como o sistema é causal, a RDC está à direita do po
mais à direita. Como é estável, inclui o eixo $j\omega$.



$$H(s) = \frac{s+2}{(s+3)(s+1)}$$

Polos em $s = \{-1, -3\}$

zeros em $s = \{-2, \infty\}$

$$H(s) = \frac{A_1}{s+1} + \frac{A_2}{s+3} \Rightarrow H(s) = \frac{1/2}{s+1} + \frac{1/2}{s+3}$$

$$\left. \frac{s+2}{s+3} \right|_{s=-1} = \frac{1}{2}$$

$$h(t) = \frac{1}{2}\left(e^{-t} + e^{-3t}\right) u(t)$$

$$\left. \frac{s+2}{s+1} \right|_{s=-3} = \frac{-1}{-2} = 1/2$$