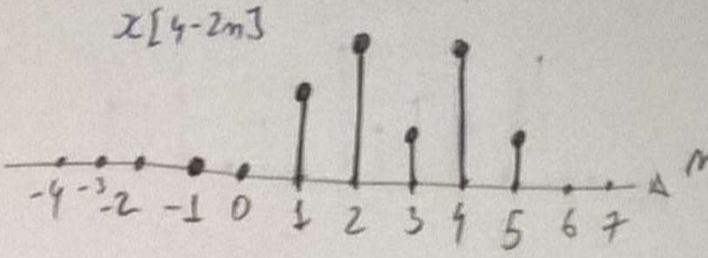
GABARITO (1EE 16.2) Sinais e Sistemas

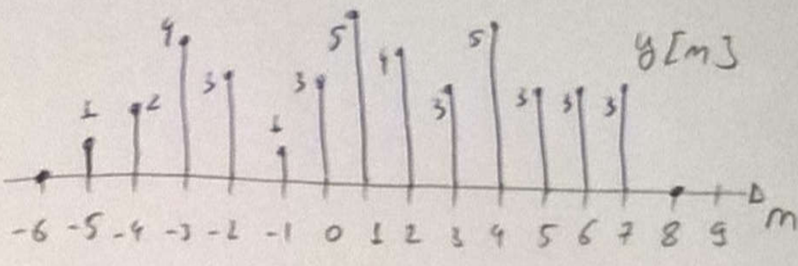
① a) $x[4-2m] = x[-2(m-2)]$



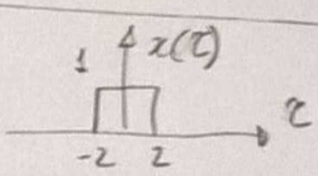
b) $x[m] * (u[m-1] - u[m-3])$

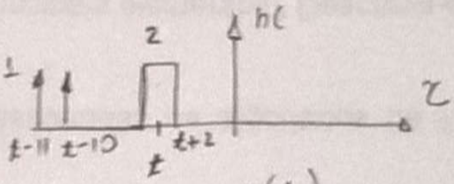
$x[m] * (\delta[m-1] + \delta[m-2])$

$x[m-1] + x[m-2] = y[m]$



② $x(t) = \begin{cases} 1, & -2 \le t \le 2 \\ 0, & c.c. \end{cases}$





$x(t) * \delta(t-10) = x(t-10)$

$x(t) * \delta(t-11) = x(t-11)$

$x(t) * 2\Pi\left(\frac{t}{2}\right) = y_1(t)$

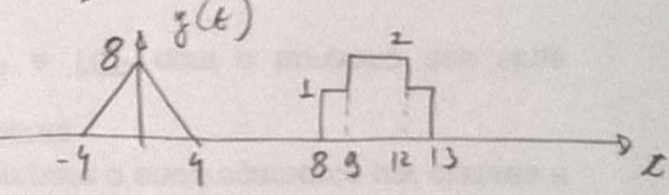
$-4 \le t \le 0$

$y_1(t) = \int_{-2}^{t+2} 2\, d\tau = 2\tau \Big|_{-2}^{t+2} = 2t+8$

$0 \le t \le 4$

$y_1(t) = \int_{t-2}^{2} 2\, d\tau = 2\tau \Big|_{t-2}^{2} = -2t+8$

b)



a) $y(t) = \begin{cases} 2t+8, & -4 \le t \le 0 \\ -2t+8, & 0 \le t \le 4 \\ 1, & 8 \le t \le 9 \\ 2, & 9 \le t \le 12 \\ 1, & 12 \le t \le 13 \\ 0, & c.c. \end{cases}$

③ a) $h[m] = [0 \ \ \frac{1}{3} \ \ \frac{2}{9} \ \ \frac{1}{9}]$ $\quad x[m] = [6 \ \ 12]$

$y[0] = x[0]h[0] = 0$

$y[1] = x[0]h[1] + x[1]h[0] = 2$

$y[2] = x[0]h[2] + x[1]h[1] = 16/3$

$y[3] = x[0]h[3] + x[1]h[2] = 10/3$

$y[4] = x[0]h[4] + x[1]h[3] = 4/3$

$y[m] = [0 \ \ 2 \ \ \frac{16}{3} \ \ \frac{10}{3} \ \ \frac{4}{3}]$

GABARITO (1EE 2016.1) Sinais e Sistemas

3 b) se $x_1[n] = 2x[n] - x[n-1]$, $y[n] = x[n] * h[n]$

como o sistema é LIT

$2x[n] * h[n] = 2y[n]$ $\qquad y_1[n] = 2y[n] - y[n-1]$

$-x[n-1] * h[n] = -y[n-1]$ $\qquad y_1[n] = [0\ 4\ \frac{26}{3}\ \frac{4}{3}\ -\frac{2}{3}\ -\frac{4}{3}]$

4 a) $a_0 = \frac{1}{2}\int_0^1 (1-t)\,dt + \frac{1}{2}\int_{-1}^0 (1+t)\,dt = \frac{1}{2}\left[t - \frac{t^2}{2}\Big|_0^1 + t + \frac{t^2}{2}\Big|_{-1}^0\right] = \frac{1}{2}$

se $w_0 = 2\pi/2 = \pi$ e $x(t)$ tem simetria par

$$a_k = \frac{2}{2}\int_0^1 (1-t)\cos(k\pi t)\,dt = -\frac{\operatorname{sen}(k\pi t)}{k\pi}\Big|_0^1 + \frac{t.\operatorname{sen}(k\pi t)}{k\pi}\Big|_0^1 + \frac{\cos(k\pi t)}{(k\pi)^2}\Big|_0^1 = \frac{(-1)^k - 1}{(k\pi)^2}$$

b) $a_k = \frac{1}{2}\int_{-1}^0 (1+t)e^{-jk\pi t}\,dt + \frac{1}{2}\int_0^1 (1-t)e^{-jk\pi t}\,dt$

$$a_k = \frac{1}{2}\left[-\frac{e^{-jk\pi t}}{jk\pi}\Big|_{-1}^0 - \frac{t\,e^{-jk\pi t}}{jk\pi}\Big|_{-1}^0 - \frac{e^{-jk\pi t}}{(jk\pi)^2}\Big|_{-1}^0 - \frac{e^{-jk\pi t}}{jk\pi}\Big|_0^1 + t.\frac{e^{-jk\pi t}}{jk\pi}\Big|_0^1 + \frac{e^{-jk\pi t}}{(jk\pi)^2}\Big|_0^1\right]$$

$$a_k = \frac{1}{2jk\pi}\left[e^0 - e^{jk\pi} - 0 - e^{jk\pi} - \frac{1}{jk\pi}\left(e^0 - e^{jk\pi}\right)\right] + \frac{1}{2jk\pi}\left[e^{-jk\pi} - e^0 + e^{-jk\pi} - 0 + \frac{1}{jk\pi}\left(e^{-jk\pi} - e^0\right)\right]$$

$$a_k = \frac{1}{2jk\pi}\left[0 - 2e^{jk\pi} + 2e^{-jk\pi} + \frac{1}{jk\pi}\left(-2 + e^{jk\pi} + e^{-jk\pi}\right)\right] = \frac{1}{jk\pi}\left[\overset{\to 0}{\cancel{2\operatorname{sen}(k\pi)}} + \frac{1}{jk\pi}\left(-2 + 2\cos(k\pi)\right)\right]$$

$$a_k = \frac{1}{(jk\pi)^2}\left(-1 + \cos(k\pi)\right) = \frac{(-1)^k - 1}{(k\pi)^2}$$

c) se $x(t) \xleftrightarrow{SF} a_k$, $\frac{d}{dt}x(t) \xleftrightarrow{SF} jw_0 a_k$

se b_k são os coeficientes de $\frac{d}{dt}x(t)$

$$b_k = j\frac{2\pi}{2}a_k = j\pi\frac{((-1)^k - 1)}{(k\pi)^2}$$