

① a) $X_1(\omega) = \int_{-\infty}^{\infty} e^{-2t} u(t-2) e^{-j\omega t} dt = \int_2^{\infty} e^{-2t} e^{-j\omega t} dt = \frac{-1}{j\omega+2} e^{-(j\omega+2)t} \Big|_{t=2}^{\infty} = \frac{1}{j\omega+2} e^{-2j\omega-4}$

b) $\text{sinc}(2t) \xrightarrow{\mathcal{F}} \frac{\pi}{2} \Pi\left(\frac{\omega}{2}\right) = \frac{\pi}{2} \cdot \begin{cases} 1, & |\omega| \leq 2 \\ 0, & \text{c.c.} \end{cases}$

$X_2(\omega) = X_1(\omega) \frac{\pi}{2} \Pi\left(\frac{\omega}{2}\right) = \begin{cases} \frac{\pi}{2} \cdot \frac{1}{j\omega+2} \cdot e^{-2j\omega-4}, & |\omega| \leq 2 \\ 0, & \text{c.c.} \end{cases}$

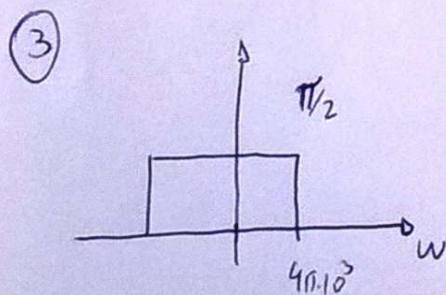
c) $X_3(\omega) = j\omega X_2(\omega) = \begin{cases} j\frac{\pi}{2} \cdot e^{-4} \cdot \frac{e^{-2j\omega} \cdot \omega}{j\omega+2}, & |\omega| \leq 2 \\ 0, & \text{c.c.} \end{cases}$

② a) $H(\omega) = \frac{5j\omega + 12}{(j\omega)^2 + 8j\omega + 15} = \frac{5j\omega + 12}{(j\omega+5)(j\omega+3)} = \frac{A_1}{j\omega+5} + \frac{A_2}{j\omega+3} = \frac{+13/2}{j\omega+5} - \frac{3/2}{j\omega+3}$

$h(t) = \frac{13}{2} e^{-5t} u(t) - \frac{3}{2} e^{-3t} u(t)$

b) $Y(\omega) = H(\omega) X(\omega) = \frac{5j\omega + 12}{(j\omega+5)^2 + (j\omega+3)} = \frac{A_1}{j\omega+5} + \frac{A_2}{j\omega+5} + \frac{A_3}{(j\omega+5)^2} = \frac{-3/4}{j\omega+5} + \frac{3/4}{j\omega+5} + \frac{13/2}{(j\omega+5)^2}$

$y(t) = \frac{-3}{4} e^{-5t} u(t) + \frac{3}{4} e^{-5t} u(t) + \frac{13}{2} t e^{-5t} u(t)$

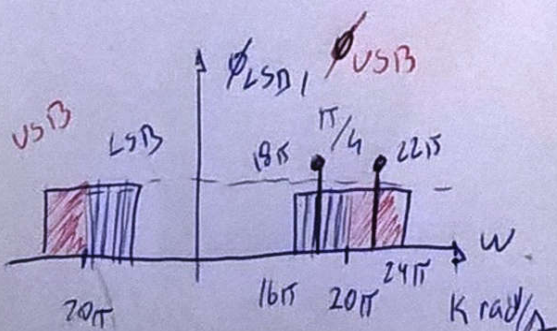


$\pi B \text{sinc}(2\pi B t) \xrightarrow{\mathcal{F}} \frac{\pi}{2} \Pi\left(\frac{\omega}{2\pi B}\right) = \begin{cases} \frac{\pi}{2}, & |\omega| \leq 4\pi \cdot 10^3 \\ 0, & \text{c.c.} \end{cases}$

$\omega_c = 20\pi \text{ Krad/s}$

$\varphi_{LSD}(t) = m(t) \cos(\omega_c t) + m_H(t) \sin(\omega_c t)$

$\varphi_{USD}(t) = m(t) \cos(\omega_c t) - m_H(t) \sin(\omega_c t)$



$m_H(t) = \frac{\pi B \sin(2\pi B t + \pi/2)}{2\pi B t} = \frac{\pi B \cos(2\pi B t)}{2\pi B t}$

$\varphi_{USB}(t) = \pi B \text{sinc}(4\pi B t) \cdot \cos(22\pi \cdot 10^3 t) \quad // \quad (\text{USB})$

$\varphi_{LSD}(t) = \pi B \text{sinc}(4\pi B t) \cdot \cos(18\pi \cdot 10^3 t) \quad // \quad (\text{LSD})$

$$4) a) k_p m(t) = 20 \sin(1000\pi t) + 10 \sin(2000\pi t)$$

$$b) k_f m(t) = \frac{d}{dt} \{ 20 \sin(1000\pi t) + 10 \sin(2000\pi t) \} = 2 \cdot 10^4 \pi \cos(10^3 \pi t) + 2 \cdot 10^4 \pi \cos(2 \cdot 10^3 \pi t)$$

$$c) \Delta f = \frac{k_f m_p}{2\pi} = \frac{4 \cdot 10^4 \pi}{2\pi} = 2 \cdot 10^4 = 20 \text{ kHz}$$

$$d) \Delta \theta = \max \{ \theta(t) \} = 20 + 10 = 30 \text{ rad}$$

$$e) B_N = 2(\Delta f + D) = 2(20 \text{ k} + 1 \text{ k}) = \del{42 \text{ kHz}} \quad 42 \text{ kHz}$$